3.27) Primero ~~busco~~ armo una base de $\mathbb{R}^4$ que contenga $[1\ 2\ -1\ -2]$, que

Puede ser: $B = \{\underbrace{[1\ 2\ -1\ -2]^T}_{v_1}, \underbrace{[0\ 1\ 0\ 0]^T}_{v_2}, \underbrace{[0\ 0\ 1\ 0]^T}_{v_3}, \underbrace{[0\ 0\ 0\ 1]^T}_{v_4}\}$

Busco una base ortogonal $B' = \{w_1, w_2, w_3, w_4\}$ por Gram-Schmidt:

$w_1 = v_1$

$w_2 = v_2 - \dfrac{\langle v_2, w_1\rangle}{\langle w_1, w_1\rangle} w_1$ I $\rightarrow \boxed{w_2 = [-1\ 3\ 1\ 2]^T}$

$w_3 = v_3 - \dfrac{\langle v_3, w_1\rangle}{\langle w_1, w_1\rangle} w_1 - \dfrac{\langle v_3, w_2\rangle}{\langle w_2, w_2\rangle} w_2$ II $\rightarrow \boxed{w_3 = [1\ 0\ 5\ -2]^T}$

$w_4 = v_4 - \dfrac{\langle v_4, w_1\rangle}{\langle w_1, w_1\rangle} w_1 - \dfrac{\langle v_4, w_2\rangle}{\langle w_2, w_2\rangle} w_2 - \dfrac{\langle v_4, w_3\rangle}{\langle w_3, w_3\rangle} w_3$ III

$\rightarrow \boxed{w_4 = [2\ 0\ 0\ 1]^T}$

Por lo tanto la base ortogonal de $\mathbb{R}^4$ hallada es:

$$\boxed{B' = \left\{ \underbrace{\begin{bmatrix} 1 \\ 2 \\ -1 \\ -2 \end{bmatrix}}_{w_1}, \underbrace{\begin{bmatrix} -1 \\ 3 \\ 1 \\ 2 \end{bmatrix}}_{w_2}, \underbrace{\begin{bmatrix} 1 \\ 0 \\ 5 \\ -2 \end{bmatrix}}_{w_3}, \underbrace{\begin{bmatrix} 2 \\ 0 \\ 0 \\ 1 \end{bmatrix}}_{w_4} \right\}}$$

CALCULOS AUX (uso PIC de $\mathbb{R}^4$)

I) $[0\ 1\ 0\ 0]^T - \frac{2}{10} \cdot [1\ 2\ -1\ -2]^T = [0\ 1\ 0\ 0]^T - \left[\frac{1}{5}\ \frac{2}{5}\ \frac{-1}{5}\ \frac{-2}{5}\right]^T = \left[\frac{-1}{5}\ \frac{3}{5}\ \frac{1}{5}\ \frac{2}{5}\right]^T$

$\times 5$

$= [-1\ 3\ 1\ 2]^T$

II) $[0\ 0\ 1\ 0]^T - \left(\left(\frac{-1}{10}\right) \cdot [1\ 2\ -1\ -2]^T\right) - \left(\frac{1}{15} \cdot [-1\ 3\ 1\ 2]^T\right) =$

$= [0\ 0\ 1\ 0]^T - \left[\frac{-1}{10}\ \frac{-1}{5}\ \frac{1}{10}\ \frac{1}{5}\right]^T - \left[\frac{-1}{15}\ \frac{1}{5}\ \frac{1}{15}\ \frac{2}{15}\right]^T = \left[\frac{1}{10}+\frac{1}{15}\ \ 0\ \ \frac{9}{10}-\frac{1}{15}\ \ \frac{-1}{5}-\frac{2}{15}\right]^T =$

$= \left[\frac{1}{6}\ 0\ \frac{5}{6}\ \frac{-1}{3}\right]^T \times 6 = [1\ 0\ 5\ -2]^T$

$$\text{III} \quad [0\,0\,0\,1]^T - \left(\frac{-1}{5} \cdot [1\;2\;-1\;-2]^T\right) - \left(\frac{2}{15} \cdot [-1\;3\;1\;2]^T\right) - \left(\frac{-1}{15} \cdot [1\;0\;5\;-2]^T\right) =$$

$$= [0\,0\,0\,1]^T - \left[\frac{-1}{5}\;\; \frac{-2}{5}\;\; \frac{1}{5}\;\; \frac{2}{5}\right]^T - \left[\frac{-2}{15}\;\; \frac{2}{5}\;\; \frac{2}{15}\;\; \frac{4}{15}\right]^T - \left[\frac{-1}{15}\;\; 0\;\; \frac{-1}{3}\;\; \frac{2}{15}\right]^T =$$

$$= \left[\frac{1}{5} + \frac{2}{15} + \frac{1}{15}\quad \frac{2}{5} - \frac{2}{5}\quad \frac{-1}{5} - \frac{2}{15} + \frac{1}{3}\quad \frac{3}{5} - \frac{4}{15} - \frac{2}{15}\right]^T =$$

$$= \left[\frac{2}{5}\;\; 0\;\; 0\;\; \frac{1}{5}\right]^T \times 5 = [2\;\; 0\;\; 0\;\; 1]^T$$